Learning Objectives

- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Heating of Motor or Temperature Rise
- Equation for Heating of Motor
- Heating Time Constant
- Equation for Cooling of Motor or Temperature Fall
- Cooling Time Constant
- Heating and Cooling Curves
- Load Equalization
- Use of Flywheels
- Flywheel Calculations
- Load Removed (Flywheel Accelerating)
- Choice of Flywheel

CHAPTER 45

RATING AND SERVICE CAPACITY

Generator converts mechanical energy into electrical energy using electromagnetic induction
45.1. Size and Rating

The factors which govern the size and rating of a motor for any particular service are its maximum temperature rise under given load conditions and the maximum torque required. It is found that a motor which is satisfactory from the point of view of maximum temperature rise usually satisfies the requirement of maximum torque as well. For class-A insulation, maximum permissible temperature rise is 40°C whereas for class-B insulation, it is 50°C. This temperature rise depends on whether the motor has to run continuously, intermittently or on variable load.

Different ratings for electrical motors are as under:

1. **Continuous Rating.** It is based on the maximum load which a motor can deliver for an indefinite period without its temperature exceeding the specified limits and also possessing the ability to take 25% overload for a period of time not exceeding two hours under the same conditions.

   For example, if a motor is rated continuous 10 KW, it means that it is capable of giving an output of 10 KW continuously for an indefinite period of time and 12.5 KW for a period of two hours without its temperature exceeding the specified limits.

2. **Continuous Maximum Rating.** It is the load capacity as given above but without overload capacity. Hence, these motors are a little bit inferior to the continuous-rated motors.

3. **Intermittent Rating.** It is based on the output which a motor can deliver for a specified period, say one hour or ½ hour or ¼ hour without exceeding the temperature rise.

   This rating indicates the maximum load of the motor for the specified time followed by a no-load period during which the machine cools down to its original temperature.

45.2. Estimation of Motor Rating

Since primary limitation for the operation of an electric motor is its temperature rise, hence motor rating is calculated on the basis of its average temperature rise. The average temperature rise depends on the average heating which itself is proportional to the square of the current and the time for which the load persists.

For example, if a motor carries a load \( L_1 \) for time \( t_1 \) and load \( L_2 \) for time \( t_2 \) and so on, then

\[
\text{Average heating} \propto L_1^2 t_1 + L_2^2 t_2 + \ldots + L_n^2 t_n
\]

In fact, heating is proportional to square of the current but since load can be expressed in terms of the current drawn, the proportionality can be taken for load instead of the current.

\[
\therefore \text{size of the motor} = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + \ldots + L_n^2 t_n}{t_1 + t_2 + \ldots + t_n}}
\]

Generally, load on a motor is expressed by its load cycle. Usually, there are periods of no-load in the cycle. When motor runs on no-load, heat generated is small although heat dissipation continues at the same rate as long as the machine is running. Hence, there is a difference in the heating of a motor running at no-load and when at rest. It is commonly followed practice in America to consider the period at rest as one – third while calculating the size of motor. It results in giving a higher motor rating which is advantageous and safe.

**Example 45.1** An electric motor operates at full-load of 100 KW for 10 minutes, at \( \frac{3}{4} \) full load for the next 10 minutes and at \( \frac{1}{2} \) load for next 20 minutes, no-load for the next 20 minutes and this cycle repeats continuously. Find the continuous rating of the suitable motor.

**Solution.**

Size of the motor required

\[
= \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + 20}}
\]

\[= 61 \text{ KW}\]
According to American practice, we will consider the period of rest as (20/3) minutes. In that case, the motors size is

$$= \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + (20/3)}}$$

$$= 66 \text{ KW}$$

**Example 45.2.** An electric motor has to be selected for a load which rises uniformly from zero to 200 KW in 10 minutes after which it remains constant at 200 KW for the next 10 minutes, followed by a no-load period of 15 minutes before the cycle repeats itself. Estimate a suitable size of continuously rated motor.

**Solution.**

$$\text{Motor size } = \sqrt{(200/2)^2 \times 10 + (200)^2 \times 10 + 0 \times 15} \div (10 + 10 + (15 \times 1/3))$$

$$= 140 \text{ KW}$$

According to American practice, no-load has been taken as one third.

**Example 45.3.** A certain motor has to perform the following duty cycle:

- 100 KW for 10 minutes
- 50 KW for 8 minutes
- No-load for 5 minutes
- No-load for 4 minutes

The duty cycle is repeated indefinitely. Draw the curve for the load cycle. Assuming that the heating is proportional to the square of the load, determine suitable size of a continuously-rated motor.

**Solution.**

As explained above, heating is proportional to the square of the current and hence, to the square of the load.

$$\therefore \text{ size of the continuously-rated motor } = \sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}}$$

$$= 66.67 \text{ kW}$$

Hence, the motor of 70 KW would be adequate. The curve of the load cycle is shown in Fig. 45.1

The ultimate usefulness of the above factors is to select a motor of as small a size as possible compatible with temperature rise and to ensure that the motor has ample overload torque to cater for maximum-load conditions. Obviously, over-motoring of any industrial drive will result in a waste of electrical energy, a low power factor and unnecessarily high capital cost for the motor and control gear.

**45.3. Different Types of Industrial Loads**

The three different types of industrial loads under which electric motors are required to work are as under:

- (i) continuous load
- (ii) intermittent load
- (iii) variable or fluctuating load

The size of the motor depends on two factors. Firstly, on the temperature rise which, in turn,
will depend on whether the motor is to operate on continuous, intermittent or variable load. Secondly, it will depend on the maximum torque to be developed by the motor. Keeping in mind the load torque requirements, the rating of the motor will be decided by the load conditions as described below.

(i) Continuous Load. In such cases, the calculation of motor size is simpler because the loads like pumps and fans require a constant power input to keep them operating. However, it is essential to calculate the KW rating of the motor correctly. If the KW rating of the motor is less than what is required, the motor will overheat and consequently burn out. If, on the other hand, KW rating is more than what is needed by the load, the motor will remain cool but will operate at lower efficiency and power.

(ii) Intermittent Loads. Such loads can be of the following two types:

(a) In this type of load, motor is loaded for a short time and then shut of for a sufficient by long time, allowing the motor to cool down to room temperature as shown in Fig. 45.2. In such cases, a motor with a short time rating is used as in a kitchen mixie.

Torque motors are designed to provide maximum torque at locked rotor or near stalled conditions. Their applications are in servo and positioning systems, tension reels, automatic door openers, and filament winding equipment.
(b) In this type of load, motor is loaded for a short time and then it is shut off for a short time. The shut off time is so short that the motor cannot cool down to the room temperature as shown in Fig.45.3. In such cases, a suitable continuous or short-time rated motor is chosen which, when operating on a given load cycle, will not exceed the specified temperature limit.

(iii) Variable Loads. In the case of such loads, the most accurate method of selecting a suitable motor is to draw the heating and cooling curves as per the load fluctuations for a number of motors. The smallest size motor which does not exceed the permitted temperature rise when operating on the particular load cycle should be chosen for the purpose.

However, a simpler but sufficiently accurate method of selection of a suitable rating of a motor is to assume that heating is proportional to the square of the current and hence the square of the load. The suitable continuous rating of the motor would equal the r.m.s. value of the load current.

**Example 45.4.** A motor has to perform the following duty cycle

| 100 H.P. | For 10 min |
| No Load | " 5 min |
| 60 H.P. | " 8 min |
| No Load | " 4 min |

which is repeated infinitely. Determine the suitable size of continuously rated motor.

**Solution.**

\[
\text{R.M.S. H.P.} = \sqrt{\frac{1}{\text{Time for one cycle}} \int HP^2 \, dt}
\]

\[
\text{R.M.S. H.P.} = \sqrt{\sum \frac{HP^2 \times \text{time}}{\text{Time for one cycle}}}^{\frac{1}{2}}
\]

\[
= \sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}} = 69.07 \text{ H.P.}
\]

\[
= 75 \text{ H.P. motor can be used.}
\]
Example 45.5. A motor working in a coal mine has to exert power starting from zero and rising uniformly to 100 H.P. in 5 min after which it works at a constant rate of 50 H.P. for 10 min. Then, a no load period of 3 min. The cycle is repeated indefinitely, estimate suitable size of motor.

Solution.

(a) For time period : 0 - 5 min

\[ y = mx + c \]

Slope = \( \frac{(100 - 0)}{5} \)

\[ m = 20 \text{ HP/min} \]

\[ \therefore y = 20x + 0 \]

(b) For total time period : 0 - 18 min

R.M.S. H.P.²

\[ \Rightarrow \text{H.P.}² = \frac{1}{2} \int_0^{18} (20x)^2 \text{ dx} + 25000 = \frac{400x^3}{3} \bigg|_0^{18} + 25000 \]

\[ \therefore \text{H.P.} = \sqrt{\frac{41666.67}{18}} \approx 48.11 \text{ H.P.} = 50 \text{ H.P. motor can used} \]

or Same problem can be solved by Simpson’s 1/3rd Rule of Integration

\[ \text{H. P.} = \sqrt{\frac{1}{3} \times 100^2 	imes 5 + 50^2 	imes 10} \]

\[ \text{H. P.} = 48.11 \text{ H.P. ; } \therefore \text{H. P. } \approx 50 \text{ H.P.} \]

Example 45.6. A motor has following duty cycle

Load rising from 200 to 400 H.P. - 4 min.
Uniform load 300 H.P. - 2 min.
Regenerative braking - H.P. returned to supply from 50 to zero - 1 min.
Remaining idle for - 1 min.

Estimate suitable H.P. rating of the motor. Motor can be used.

[Nagpur University Winter 1994]
Solution.

\[
H. \ P. = \sqrt{\frac{\frac{1}{3} (H_1^2 + H_1 H_2 + H_2^2) t_1 + H_1^2 t_2 + \frac{1}{3} H_2^2 t_3}{8}}
\]

\[
= \sqrt{\frac{\frac{1}{3} (200^2 + 200 \times 400 + 400^2) \times 4 + 300^2 \times 2 + \frac{1}{3} 50^2 \times 1}{8}}
\]

\[
= \sqrt{\frac{1662500}{24}} = 263 \text{ H. P.}
\]

**Note.** During regenerative braking, even though H.P. is returned to line, machine will be carrying current. So far heating is concerned, it is immaterial whether machine is taking current from or giving current to line.

This problem can be solved by another method as follows:-

(a) For time period : 0 - 4 min

\[
\rightarrow \int_{0}^{4} (50x + 200)^2 \ dx = \int_{0}^{4} (250x^2 + 20000x + 40000) \ dx
\]

\[
= 2500 \left[ \frac{x^3}{3} \right]_0^4 + 20000 \left[ \frac{x^2}{2} \right]_0^4 + 40000x
\]

\[
= 2500 \left[ 4^3 - 0^3 \right] + 20000 \left[ 4^2 - 0^2 \right] + 40000 \times 4 = 373333.3 \text{ H.P.}
\]

(b) For time period : 4 - 6 min

\[
\rightarrow (300)^2 \times 2 = 180000 \text{ H.P.}
\]

(c) For time period : 6 - 7 min

\[
\rightarrow \int_{0}^{1} (50x)^2 \ dx = 2500 \left[ \frac{x^3}{3} \right]_0^1 = \frac{2500}{3} = 833.33 \text{ H.P}
\]

\[
\therefore \ R.M.S. \text{ H.P.} = \sqrt{\frac{373333.33 + 180000 + 833.33}{8}} = 263.1 \text{ H.P.}
\]

\[
\geq 300 \text{ H.P. motor will be suitable}
\]
Example 45.7. The load cycle of a motor for 15 min. in driving some equipment is as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 5 min</td>
<td>30 H.P.</td>
</tr>
<tr>
<td>5 - 9 min</td>
<td>No Load</td>
</tr>
<tr>
<td>9 - 12 min</td>
<td>45 H.P.</td>
</tr>
<tr>
<td>12 - 15 min</td>
<td>No Load</td>
</tr>
</tbody>
</table>

The load cycle is repeated indefinitely. Suggest a suitable size of continuously rated motor.

Solution.

\[
R.M.S. \quad H.P. = \left[ \frac{\sqrt{30^2 \times 5 + 45^2 \times 3}}{15} \right]^{\frac{1}{2}}
\]

\[
= 26.55 \quad H.P.
\]

\[\therefore \quad R.M.S. \quad H.P. \approx 30 \quad H.P. \quad motor \quad will \quad be \quad suitable.\]

Example 45.8. A motor driving a colliery winder has the following acceleration period load cycle 0 - 15 sec.: Load rising uniformly from 0 to 1000 H.P.

Full speed period: 15 - 85 sec. Load const. at 600 H.P.

Deceleration period: 85 to 95 sec. regenerative braking the H.P. returned uniformly from 200 to 0 H.P.

95 - 120 sec.: Motor stationary.

Estimate the size of continuously rated motor.

Solution.

\[
R.M.S. \quad H.P. = \left[ \frac{\frac{1}{3} \times (1000)^2 \times 15 + 600^2 \times 70 + \frac{1}{3} \times (200)^2 \times 10}{120} \right]^{\frac{1}{2}}
\]

\[
= 502 \quad H.P.
\]

\[\cong 505 \quad H.P. \quad motor \quad can \quad be \quad used.\]

45.4. Heating of Motor or Temperature Rise

The rise in temperature of a motor results from the heat generated by the losses and an expression for this temperature rise is obtained by equating the rate at which heat is being generated by these losses to the rate at which heat is being absorbed by the motor for raising the temperature of motor and in dissipation from the surfaces exposed to cooling media.

So long as the temperature of machine rises, the generated heat will be stored in body and the rest will be dissipated to cooling medium depending upon the temperature difference. This is called as unstable or transient situation.

If the temperature of body rises, it has to store heat. The amount of heat i.e. stored depends upon the heat capacity of the body. If the temperature of the machine remains constant i.e. it doesn’t rise, then no further storage of heat takes place and all the heat i.e. generated must be dissipated. So rate of heat generation in motor equals rate of heat dissipation from the cooling surface. This is called a stable situation.

45.5. Equation for Heating of Motor

Let,

\[W \rightarrow \quad \text{Heat generated in motor due to powerloss in watts.}\]

\[G \rightarrow \quad \text{Weight of motor (kg)}\]
**Rating and Service Capacity**

\[ S \rightarrow \text{Average specific heat in (Watt - Sec.) to raise the temperature of unit weight through 1°C.} \]

\[ G \times S \rightarrow \text{Heat required to raise the temperature of motor through 1°C (Watt - Sec.)} \]

\[ \theta \rightarrow \text{Temperature rise above cooling medium in °C.} \]

\[ \theta_f \rightarrow \text{Final temperature rise in °C.} \]

\[ \lambda \rightarrow \text{Rate of heat dissipation from the cooling surface.} \]

\[ \text{[(Watts/Unit area/°C rise in temperature.) above cooling medium]} \]

\[ A\lambda \rightarrow \text{Rate of heat dissipation in Watts/°C rise in temperature for a motor.} \]

**Assumptions**

1. Loss ‘W’ remains constant during temperature rise.
2. Heat dissipation is proportional to the temperature difference between motor and cooling medium.
3. Temperature of cooling medium remains constant.

{Rate of heat generation in motor} = {Rate of heat absorption by the motor} + {Rate of heat dissipation from cooling surface}

\[
W = GS \frac{d\theta}{dt} + A\lambda \theta
\]

or

\[
W - A\lambda \theta = GS \frac{d\theta}{dt}
\]

\[
\frac{W}{A\lambda} - \theta = GS \frac{d\theta}{A\lambda} dt
\]

\[
\frac{d\theta}{\left(\frac{W}{A\lambda} - \theta\right)} = \frac{GS}{A\lambda} dt
\]

By integrating,

\[
\log_e \left(\frac{W}{A\lambda} - \theta\right) = \frac{A\lambda}{GS} t + C \quad \text{..... (2)}
\]

At \[ t = 0 \], \[ \theta = \theta_1 \] [Initial temperature rise i.e. difference between the temperature of cooling medium and temperature of motor, during starting]

If starting from cold position, \[ \theta_1 = 0 \]

Substituting the values of \( t \) and \( \theta \) in above equation.

\[
C = \log_e \left(\frac{W}{A\lambda} - \theta_1\right)
\]

\[
\therefore \ (2) \Rightarrow \log_e \left(\frac{W}{A\lambda} - \theta_1\right) = -\frac{A\lambda}{GS} t
\]

\[
\frac{W}{A\lambda} - \theta_1 = e^{-\frac{A\lambda}{GS} t}
\]

by taking antilog,
\[ \theta = W \left( \frac{W}{A\lambda} - \theta_i \right) e^{\frac{A\lambda}{GS}} \]  

\[ \therefore \quad \theta = W \left( \frac{W}{A\lambda} - \theta_i \right) e^{\frac{A\lambda}{GS}} \]  

When, the final temperature rise of \( \theta_f \) is reached, all the heat generated is dissipated from the cooling surface so that,

equation (1) becomes \( W = A\lambda \theta_f \) or \( \theta_f = \frac{W}{A\lambda} \)

And

\[ \frac{GS}{A\lambda} = \text{Heating time constant} \]

\[ \therefore \quad \frac{A\lambda}{GS} = \frac{1}{T} \]

Then equation (3) becomes;

\[ \theta = \theta_f - (\theta_f - \theta_i) e^{\frac{-t}{T}} \]

If starting from cold, then \( \theta_i = 0 \)

\[ \therefore \quad \theta = \theta_f (1 - e^{\frac{-t}{T}}) \]

45.6. Heating Time Constant

Heating time constant of motor is defined as the time required to heat up the motor upto 0.633 times its final temperature rise.

\[ \theta = (1 - e^{-UT}) \]

At \( t = T \), \( \theta = 0.633 \theta_f \)

After time \( t = T \) \( \theta \) reaches to 63.3 % of \( \theta_f \)

\( t = 2T \) \( \theta \) reaches to 86.5 % of \( \theta_f \)

\( t = 3T \) \( \theta \) reaches to 95 % of \( \theta_f \)

\( t = 4T \) \( \theta \) reaches to 98.2 % of \( \theta_f \)

\( t = 5T \) \( \theta \) reaches to 99.3 % of \( \theta_f \)

\( T \) = Heating time constant.

= 90 min for motors upto 20 H.P.

= 300 min for larger motors.

45.7. Equation for Cooling of Motor or Temperature Fall

If rate of heat generation is less than rate of heat dissipation, cooling will take place.

\[ \therefore \quad \{ \text{Rate of heat generation in motor} \} + \{ \text{Rate of heat absorption by motor} \} = \{ \text{Rate of heat dissipation from cooling surface} \} \]

\[ W + GS \frac{d\theta}{dt} = A\lambda \theta \]  \quad where \( \lambda' \) = Rate of heat dissipation during cooling surface

\[ W - A\lambda' \theta = -GS \frac{d\theta}{dt} \quad \Rightarrow \quad \frac{W}{A\lambda'} \theta = -\frac{GS}{A\lambda'} \frac{d\theta}{dt} \]

\[ \theta - \frac{W}{A\lambda'} \theta = -\frac{GS}{A\lambda'} \frac{d\theta}{dt} \quad \Rightarrow \quad \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \frac{dt}{ GS \frac{d\theta}{A\lambda'}} \]
Rating and Service Capacity

\[ \int \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \int \frac{dt}{GS} \]

\[ \log_e \left( \frac{\theta - \frac{W}{A\lambda'}}{\theta_0 - \frac{W}{A\lambda'}} \right) = -\frac{A\lambda'}{GS} t + C \]

At \( t = 0 \) let \( \theta = \theta_0 \). Difference of temperature between cooling medium and motor (Temperature rise at which cooling starts.)

\[ C = \log_e \left( \theta_0 - \frac{W}{A\lambda'} \right) \]

Put this value of \( C \) in the above equation.

\[ \log_e \left( \frac{\theta - \frac{W}{A\lambda'}}{\theta_0 - \frac{W}{A\lambda'}} \right) = -\frac{A\lambda'}{GS} t \]

If \( \theta_f \) is final temperature drop (above that of cooling medium), then at this temperature whatever heat is generated will be dissipated.

\[ W = A\lambda' \theta_f' \Rightarrow \theta_f' = \frac{W}{A\lambda'} \]

\[ \log_e \left( \frac{\theta - \theta_f'}{\theta_0 - \theta_f'} \right) = -\frac{t}{T_f} \quad \text{Where } T_f \text{ is cooling time constant } = \frac{GS}{A\lambda'} \]

\[ \frac{\theta - \theta_f'}{\theta_0 - \theta_f'} = e^{-\frac{t}{T_f}} \Rightarrow \left( \theta - \theta_f' \right) = \left( \theta_0 - \theta_f' \right) e^{-\frac{t}{T_f}} \]

\[ \theta = \theta_f' + \left( \theta_0 - \theta_f' \right) e^{-\frac{t}{T_f}} \]

If motor is disconnected from supply, there will be no losses taking place and so final temperature reached will be ambient temperature. Hence \( \theta_f' = 0 \) (\( \therefore W = 0 \))

\[ \theta = \theta_0 \cdot e^{-\frac{t}{T_f}} \]

If \( t = T_f \), then \( \theta = \theta_0 \cdot e^{-1} \Rightarrow \theta = \frac{\theta_0}{e} = 0.368 \theta_0 \); \( \therefore \theta = 0.368 \theta_0 \)

45.8. Cooling Time Constant

Cooling time constant is defined as the time required to cool machine down to 0.368 times the initial temperature rise above ambient temperature.

By putting different values of \( T_f \) in \( \theta = \theta_0 \cdot e^{-\frac{t}{T_f}} \)

\[ \therefore \text{After time } t = T_f \text{ } \theta \text{ has fallen to } 36.8\% \text{ of } \theta_0 \]

\[ t = 2T_f \text{ } \theta \text{ has fallen to } 13.5\% \text{ of } \theta_0 \]

\[ t = 3T_f \text{ } \theta \text{ has fallen to } 5\% \text{ of } \theta_0 \]
\[ t = 4T' \quad \theta \text{ has fallen to } 1.8\% \text{ of } \theta_0 \]
\[ t = 5T' \quad \theta \text{ has fallen to } 0.7\% \text{ of } \theta_0 \]

45.9. Heating and Cooling Curves

(a) Motor continuously worked on Full Load.

![Fig. 45.7](image)

Motor reaches final temperature rise and then cooling is carried out to ambient temperature.

(b) Motor Run for short time

![Fig. 45.8](image)

Temperature rise is less than maximum permissible value and the motor cooled to ambient temperature.

(c) Cooling period not sufficient to cool down the motor to its ambient temperature.

![Fig. 45.9](image)

* For intermittent loads, a motor of smaller rating can be used without exceeding maximum permissible temperature rise.
Example 45.9. A 40 KW motor when run continuously on full load, attains a temperature of 35°C, above the surrounding air. Its heating time constant is 90 min. What would be the 1/2 hour rating of the motor for this temperature rise? Assume that the machine cools down completely between each load period and that the losses are proportional to square of the load.

Solution.

Let ‘P’ KW be the ½ hour rating of the motor

\[ \theta_f \] – Final temperature rise at P K W
\[ \theta_f' \] – Final temperature rise at 40 KW

\[ \therefore \text{Losses at } P \text{ KW} \propto P^2 \]

\[ \text{Losses at } 40 \text{ K W} \propto 40^2 \]

\[ \Rightarrow \frac{\theta_f}{\theta_f'} = \left( \frac{P}{40} \right)^2 \therefore \theta_f = \left( \frac{P}{40} \right)^2 \theta_f' \]

As the machine cools down completely, for ‘P’ KW the equation will be

\[ \theta = \theta_f \left(1 - e^{\frac{1}{12}}\right) \]

\[ \theta_f = \left( \frac{P}{40} \right)^2 \times 35 \]

\[ \Rightarrow 35 = \left( \frac{P}{40} \right)^2 \times 35 \left(1 - e^{\frac{9.5}{12}}\right) \]

\[ \therefore P = 75.13 \text{ KW} \]

Example 45.10. Determine the one - hour rating of a 15 H.P. motor having heating time constant of 2 hours. The motor attains the temperature rise of 40°C on continuous run at full load. Assume that the losses are proportional to square of the load and the motor is allowed to cool down to the ambient temperature before being loaded again. [Nagpur University Summer 2001]

Solution.

Let ‘P’ H. P be one - hour rating of the motor

Losses at this load = Original losses \(\times \left( \frac{P}{15} \right)^2\).

Let \(\theta_f\) be the final temperature rise at P H.P. and \(\theta_f'\) at 15 H.P

\[ \therefore \frac{\theta_f}{\theta_f'} = \left( \frac{P}{15} \right)^2 \]

\[ \theta_f = \theta_f' \left( \frac{P}{15} \right)^2 = 40 \left( \frac{P}{15} \right)^2 \]

\[ \theta = \theta_f' \left(1 - e^{\frac{1}{12}}\right) \]

\[ 40 = 40 \left( \frac{P}{15} \right)^2 \left(1 - e^{\frac{-1}{2}}\right) \]

\[ P = 23.96 \text{ H.P.} \]

\[ P \geq 24 \text{ H.P.} \]
Example 45.11. The heating and cooling time constants of a motor are 1 hour and 2 hours respectively. Final temperature rise attained is 100°C. This motor runs at full load for 30 minutes and then kept idle for 12 min. and the cycle is repeated indefinitely. Determine the temperature rise of motor after one cycle.

[Nagpur University Winter 1997]

Solution. \[ \theta = \theta_f\left(1 - e^{-\frac{t}{T}}\right) \]
\[ = 100\left(1 - e^{-\frac{30}{60}}\right) = 39.34^\circ \]
\[ \theta = \theta_0 e^{-\frac{12}{120}} = 39.34 e^{-\frac{1}{10}} = 35.6^\circ \]

= Temperature rise of motor after 1 cycle.

Example 45.12. Calculate the maximum overload that can be carried by a 20 KW output motor, if the temperature rise is not to exceed 50°C after one hour on overload. The temperature rise on full load, after 1 hour is 30°C and after 2 hours is 40°C. Assume losses proportional to square of load.

Solution. \[ \theta = \theta_f\left(1 - e^{-\frac{1}{T}}\right) \]
\[ 30 = \theta_f\left(1 - e^{-\frac{30}{60}}\right) \]
\[ 40 = \theta_f\left(1 - e^{-\frac{40}{60}}\right) \]
\[ \frac{1 - e^{-\frac{1}{T}}}{1 - e^{-\frac{3}{6}}} = 40 \]
\[ \frac{1 - e^{-\frac{1}{T}}}{1 - e^{-\frac{1}{6}}} = 30 \]

Put \( x = e^{-\frac{1}{T}} \)

\[ \therefore \frac{1 - x^2}{1 - x} = 4 \]
\[ \frac{(1 - x)(1 + x)}{1 - x} = \frac{4}{3} \]
\[ \Rightarrow 1 + x = \frac{4}{3} \]
\[ \therefore x = \frac{1}{3} = e^{-\frac{1}{T}} \]
\[ \Rightarrow T = 0.91 \text{ hrs.} \]

To find \( \theta_f \)
\[ 30 = \theta_f\left(1 - e^{-\frac{1}{3}}\right) \]
\[ \Rightarrow 30 = \theta_f\left(1 - \frac{1}{3}\right) \Rightarrow \theta_f = 45^\circ \]

After 1 hr.
\[ 50 = \theta_f\left(1 - \frac{1}{e^{\frac{1}{3}}}\right) \]
\[ 50 = \theta_f\left(1 - \frac{1}{3}\right) \]
\[ \therefore \theta_f = 75^\circ \]

Let the maximum overload capacity of 20 KW motor is \( P \) KW

\[ \therefore \frac{\theta_f}{\theta'_f} = \frac{\text{Losses at PKW}}{\text{Original Losses}} = \left(\frac{P}{20}\right)^2 \]
Example 45.13. In a transformer the temperature rise is 25°C after 1 hour and 37.5°C after 2 hours, starting from cold conditions. Calculate its final steady temperature rise and the heating time constant. If the transformer temperature falls from the final steady value to 40°C in 1.5 hours when disconnected, calculate its cooling time constant. Ambient temperature is 30°C.

Solution. \[ \theta = \theta_f \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ 25 = \theta_f \left( 1 - e^{-\frac{1}{\tau}} \right) \quad \text{and} \quad 37.5 = \theta_f \left( 1 - e^{-\frac{2}{\tau}} \right) \]

\[ \therefore \frac{37.5}{25} = 1 - e^{-\frac{2}{\tau}} \quad \frac{1}{1 - e^{-\frac{1}{\tau}}} \quad \text{Put} \ x = e^{-\frac{1}{\tau}} \]

\[ 1.5 = 1 - x^2 \quad 1.5 = \frac{(1-x)(1+x)}{1-x} \]

\[ 1.5 = 1 + \chi \quad \therefore \chi = 0.5 \]

\[ \therefore e^{-\frac{1}{\tau}} = 0.5 \quad \therefore T = 1.44 \text{ hrs} \]

\[ 25 = \theta_f \left( 1 - e^{-\frac{1}{\tau}} \right) \quad \Rightarrow \quad 25 = \theta_f \left( e^{-\frac{1}{1.44}} \right) \quad \Rightarrow \quad \theta_f = 50°C \]

Cooling: Temperature rise after 1.5 hours above ambient temperature = 40 – 30 = 10°C.

\[ \therefore \text{The transformer is disconnected} \quad \theta = \theta_0 e^{-\frac{t}{\tau}} \]

\[ 10 = 50 e^{-\frac{1.5}{\tau}} \quad \therefore T' = 0.932 \text{ hrs} \]

Example 45.14. The initial temperature of machine is 45°C. Calculate the temperature of machine after 1.2 hours, if its final steady temperature rise is 85°C and the heating time constant is 2.4 hours. Ambient temperature is 25°C.

Solution.

\[ \theta = \theta_t - (\theta_t - \theta_0) e^{-\frac{t}{\tau}} \quad \theta = 85 - (85 - 20) e^{-\frac{1.2}{2.4}} \]

\[ \theta = 45.54°C \quad \text{Temperature rise above cooling medium} \]

\[ \therefore \text{Temperature of machine after 1.2 hours is} \quad = 45.54 + 25 = 70.54°C \]

Example 45.15. The following rises were observed in a temperature rise test on a D.C. machine at full loads.

<table>
<thead>
<tr>
<th>After</th>
<th>1 hour</th>
<th>15°C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 hours</td>
<td>25°C</td>
</tr>
</tbody>
</table>

Find out (i) Final steady temperature rise and time constant.  
(ii) The steady temperature rise after 1 hour at 50% overload, from cold.

Assume that the final temperature rise on 50% overload is 90°C.

[Nagpur University Summer 1998]
Solution. \( \theta = \theta_f \left( 1 - e^{-\frac{t}{T}} \right) \) as motor is starting from cold.

\[ 15 = \theta_f \left( 1 - e^{-\frac{3}{T}} \right) \quad \text{and} \quad 25 = \theta_f \left( 1 - e^{-\frac{2}{T}} \right) \]

\[ \therefore \quad \frac{25}{15} = \frac{\theta_f \left( 1 - e^{-\frac{2}{T}} \right)}{\theta_f \left( 1 - e^{-\frac{1}{T}} \right)} \]

\[ \therefore \quad e^{-\frac{1}{T}} = \frac{25}{15} - 1, \quad e^{-\frac{2}{T}} = \frac{2}{3} \]

\[ \therefore \quad T = 2.466 \text{ hours,} \quad 15 = \theta_f \left( 1 - e^{-\frac{1}{T}} \right) \]

by putting value of \( T \),

\[ 15 = \theta_f \left( 1 - e^{-\frac{1}{2.466}} \right) \]

\[ \Rightarrow \quad \theta_f = 45 \degree C \]

(ii) On 50\% overload \( \theta_f = 90 \degree C \)

\[ \therefore \quad \text{Final temperature rise after 1 hour at 50\% overload is} \quad \theta = \theta_f \left( 1 - e^{-\frac{1}{T}} \right) \]

\[ \theta = 90 \left( 1 - e^{-\frac{1}{2.466}} \right) = 30 \degree C \]

45.10. Load Equalization

If the load fluctuates between wide limits in space of few seconds, then large peak demands of current will be taken from supply and produce heavy voltage drops in the system. Large size of conductor is also required for this.

Process of smoothing out these fluctuating loads is commonly referred to as load equalization and involves storage of energy during light load periods which can be given out during the peak load period, so that demand from supply is approximately constant. Tariff is also affected as it is based on M.D. (Maximum Demand)

For example, in steel rolling mill, when the billet is in between the rolls it is a peak load period and when it comes out it is a light load period, when the motor has to supply only the friction and internal losses, as shown in figure 45.10.
45.11. Use Of Flywheels

The method of Load Equalization most commonly employed is by means of a flywheel. During peak load period, the flywheel decelerates and gives up its stored kinetic energy, thus reducing the load demanded from the supply. During light load periods, energy is taken from supply to accelerate flywheel, and replenish its stored energy ready for the next peak. Flywheel is mounted on the motor shaft near the motor. The motor must have drooping speed characteristics, that is, there should be a drop in speed as the load comes to enable flywheel to give up its stored energy. When the Ward-Leonard system is used with a flywheel, then it is called as Ward-Leonard Ilgner control.

45.12. Flywheel Calculations

The behaviour of flywheel may be determined as follows.

Fly wheel Decelerating :: (or Load increasing)

Let

\[ T_L \rightarrow \text{Load torque assumed constant during the time for which load is applied in kg-m} \]
\[ T_f \rightarrow \text{Torque supplied by flywheel in kg-m} \]
\[ T_o \rightarrow \text{Torque required on no load to overcome friction internal losses etc., in kg-m} \]
\[ T_m \rightarrow \text{Torque supplied by the motor at any instant, in kg-m} \]
\[ \omega_o \rightarrow \text{No Load speed of motor in rad/sec.} \]
\[ \omega \rightarrow \text{Speed of motor at any instant in rad/sec.} \]
\[ s \rightarrow \text{motor slip speed (} \omega_o - \omega \text{) in rad/sec.} \]
\[ I \rightarrow \text{Moment of inertia of flywheel in kg-m}^2 \]
\[ g \rightarrow \text{Acceleration due to gravity in } \text{m/sec}^2 \]
\[ t \rightarrow \text{time in sec.} \]

When the flywheel deaccelerates, it gives up its stored energy.

\[ T_m = T_L - T_f \quad \text{or} \quad T_L = T_m + T_f \quad \text{..... (1)} \]

Energy stored by flywheel when running at speed ' \omega ' is \( \frac{1}{2} I \omega^2/g \).

If speed is reduced from \( \omega_o \) to \( \omega \).

The energy given up by flywheel is

\[ \frac{1}{2} I \left( \omega_o^2 - \omega^2 \right) \]
\[ \frac{1}{2} I \left( \omega_o + \omega \right) \left( \omega_o - \omega \right) \quad \text{..... (2)} \]
\[ \frac{\omega_o + \omega}{2} \] = mean speed. Assuming speed drop of not more than 10%., this may be assumed equal to \( \omega \).

\[ \therefore \left( \frac{\omega_o + \omega}{2} \right) = \omega \quad \text{Also} \quad (\omega_o - \omega) = s \]

\[ \therefore \text{From equation (2), Energy given up } = \frac{I}{g} \omega s \]

\[ \text{Power given up } = \frac{I}{g} \omega \frac{ds}{dt} \]

but Torque = \( \frac{Power}{\omega} \)
Torque supplied by flywheel.

\[ T_f = \frac{1}{g} \frac{ds}{dt} \]

\[ \therefore \text{From equation (1)}, \quad T_m = T_L - \frac{1}{g} \frac{ds}{dt} \]

For values of slip speed upto 10% of No-load speed, slip is proportional to Torque or

\[ s = K T_m \]

\[ \therefore T_m = T_L - \frac{1}{g} K \frac{dT_m}{dt} \]

This equation is similar to the equation for heating of the motor

\[ W - A \lambda \theta = \text{G.S.} \quad \frac{d\theta}{dt} \]

\[ i.e. \quad (T_L - T_m) = \frac{1}{K} \frac{dT_m}{dt} \Rightarrow \frac{g}{1K} = \frac{dT_m}{(T_L - T_m)} \]

By integrating both sides.

\[ -\ln(T_L - T_m) = \frac{tg}{IK} + C_1 \quad \ldots (3) \]

At \( t = 0 \), when load starts increasing from no-load \( i.e. \ T_m = T_o \)

Hence, at \( t = 0 \) \( T_m = T_o \)

\[ \therefore C_1 = -\ln(T_L - T_o) \]

By substituting the value of \( C_1 \) above, in equation (3) – in \( (T_L - T_m) = \frac{tg}{IK} - \ln(T_L - T_0) \)

\[ \therefore \ln \left( \frac{T_L - T_m}{T_L - T_0} \right) = -\frac{tg}{IK} \Rightarrow \frac{T_L - T_m}{T_L - T_0} = e^{\frac{-tg}{IK}} \]

\[ \Rightarrow (T_L - T_m) = (T_L - T_0) e^{-\frac{tg}{IK}} \quad \therefore T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}} \]

If the Load torque falls to zero between each rolling period, then \( T_m = T_L - \left(1 - e^{-\frac{tg}{IK}}\right) \) \( \therefore T_0 = 0 \)

45.13. Load Removed (Flywheel Accelerating)

Slip speed is decreasing and therefore \( \frac{ds}{dt} \) is negative

\[ T_m = T_0 + T_f = T_0 - \frac{1}{g} \frac{ds}{dt} \Rightarrow T_0 - T_m = \frac{1}{g} K \frac{dT_m}{dt} \]

\[ \Rightarrow \frac{g}{IK} = \frac{dT_m}{T_0 - T_m} \]

After integrating both sides,

\[ -\ln(T_0 - T_m) = \frac{tg}{IK} + C \quad \text{At} \ t = 0, \ T_m = T_0 \] motor torque at the instant, when load is removed

\[ \therefore C = -\ln(T_0 - T_m) \] Putting this value of \( C \) in the above equation

\[ -\ln(T_0 - T_m) = \frac{tg}{IK} - \ln(T_0 - T_m) \]
\[ \ln \left( \frac{T_0 - T_m}{T_0 - T_m'} \right) = -\frac{t_0}{K} \]

\[ \therefore T_0 - T_m = (T_0 - T_m') e^{-\frac{t_0}{K}} \]

\[ \therefore T_m = T_0 + (T_m' - T_0) e^{-\frac{t_0}{K}} \]

Where \( T_m' \) = the motor torque, at the instant the load is removed.

45.14. Choice of Flywheel

There are two choices left for selecting a flywheel to give up its maximum stored energy:

1. Large drop in speed and small flywheel (But with this the quality of production will suffer, since a speed drop of 10 to 15% for maximum load is usually employed).

2. Small drop in speed and large flywheel. (This is expensive and creates additional friction losses. Also design of shaft and bearing of motor is to be modified.) So compromise is made between the two and a proper flywheel is chosen.

Example 45.16. The following data refers to a 500 H.P. rolling mill, induction motor equipped with a flywheel.

- No load speed → 40 r.p.m.
- Slip at full load (torque) → 12%
- Load torque during actual rolling → 41500 kg - m
- Duration of each rolling period → 10 sec.

Determine inertia of flywheel required in the above case to limit motor torque to twice its full load value. Neglect no load losses and assume that the rolling mill torque falls to zero between each rolling period. Assume motor slip proportional to full load torque.

[Nagpur University Summer 1996]
Solution. \[ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.189 \text{ rad/sec} \]

given, \[ T_0 = 0 \quad \quad T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}} \]
\[ t = 10 \text{ sec.} \]
\[ T_L = 41500 \text{ kg - m} \]
\[ T_m = 2 \times T_{\text{Full Load}} \]

Now \[ T_{\text{Full Load}} = \frac{500 \times 735.5}{0.88 \times 4.189} \text{ N-m.} \quad \therefore s = 12\% \]
\[ = 99765 \text{ N-m} \]
\[ = 10169.7 \text{ kg-m} \]
\[ \therefore T_m = 2 \times 10169.7 \]
\[ = 20339.5 \text{ kg} \]

\[ s = \frac{\omega_h - \omega}{\omega_h} \]
\[ = \frac{2\pi}{60} (N_0 - N) \]
\[ = \frac{2\pi}{60} (40 - 0.88(40)) \]
\[ = \frac{2\pi}{60} (4.8) = 0.503 \text{ rad/sec.} \]

\[ s = K \frac{T_{FL}}{\omega_h} \]
\[ 0.503 = K \frac{10169.7}{10} \implies K = 4.91 \times 10^{-5} \]

\[ T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}} \]
\[ \therefore T_0 = 0 \quad \therefore T_m = T_L \left(1 - e^{-\frac{tg}{IK}}\right) \]
\[ 20339.5 = 41500 \left(1 - \frac{-10 \times 981}{1 \times 4.91 \times 10^{-5}}\right) \]
\[ \therefore I = 2.9663 \times 10^6 \text{ kg - m}^2 \]

Example 45.17. A 6 pole, 50 Hz Induction Motor has a flywheel of 1200 kg - m$^2$ as moment of inertia. Load torque is 100 kg - m for 10 sec. No load period is long enough for the flywheel, to regain its full speed. Motor has a slip of 6% at a torque of 50 kg-m. Calculate
(i) Maximum torque exerted by motor
(ii) Speed at the end of deacceleration period.  

Solution. (i) \[ T_m = T_L - (T_L - T_0) e^{-\frac{tg}{IK}} \]

Assume \( T_0 = 0 \), \[ T_m = T_L \left(1 - e^{-\frac{tg}{IK}}\right) \]

\[ T_L = 100 \text{ kg - m}, \quad t = 10 \text{ sec}, \quad g = 9.81 \text{ m/sec}^2, \quad I = 1200 \text{ kg - m}^2 \quad s = \omega_h - \omega \]
Rating and Service Capacity

\[ s = KT \quad \therefore \text{slip speed} = s = \frac{2\pi}{60} (N_0 - N) \]

\[ N_s = \frac{120 f}{\rho} = \frac{120 \times 50}{6} = 1000 \text{ rpm} = N_0 \]

\[ N = 0.94 \times 1000 = 940 \text{ rpm}. \]

\[ \therefore S = \frac{2\pi}{60} (1000 - 940) = 2\pi \text{ rad / sec} \quad \text{or} \quad s = \frac{2\pi N}{60} = \frac{2\pi \times 1000 \times 0.06}{60} \]

\[ s = 2\pi = 6.283 \text{ rad/sec} \]

\[ T_m = 47.83 \text{ kg - m} \]

(ii) Slip speed \[ s = 0.04 \times 47.8 \text{ rad / sec} \]

\[ s = KT_m \]

\[ s = 0.125 \times (47.83) \]

\[ s = 5.98 \text{ rad/sec} \]

\[ s = 5.98 \times 60 = 57.5 \text{ rpm} = \text{slip speed} \]

\[ \therefore \text{Actual speed} = 1000 - 57.5 = 942.5 \text{ rpm} \]

Example 45.18. An Induction Motor equipped with a flywheel is driving a rolling mill which requires a Load Torque of 1900 N - m for 10 sec. followed by 250 N - m for 30 sec. This cycle being repeated indefinitely. The synchronous speed of motor is 750 r.p.m and it has a slip of 10% when delivering 1400 N-m Torque. The total Moment of Inertia of the flywheel and other rotating parts is 2100 kg-m\(^2\). Draw the curves showing the torque exerted by the motor and the speed for five complete cycles, assuming that initial torque is zero.  

[Nagpur University Summer 1998]

Solution.

\[ T_L = 1900 \text{ N- m for 10 sec.} \]

\[ T_L = 250 \text{ N- m for 30 sec.} \]

\[ T_0 = 0 \text{ (assumed)} \]

\[ T_s = 10\% \text{ at 1400 N - m torque} \]

\[ \text{Slip} = 10\% \text{ at 1400 N - m torque} \]

\[ \text{Slip} = 750 \times 0.1 = 75 \text{ r.p.m.} \]

\[ s = \frac{75 \times 2\pi}{60} = 7.85 \text{ rad/sec.} \]

\[ s = KT_m \quad K = \frac{S}{T_m} = \frac{7.85}{1400} = 0.0056 \]

(i) During 1st cycle :

(a) Flywheel de-accelerating :

\[ T_m = T_L - (T_L - T_0) e^{\frac{-t}{K}} \quad [\text{When torque is taken in N - m.}] \]

\[ \rightarrow \text{After 10 sec} \]
\[ T_m = 1900 - (1900 - 0) e^{-0.085 t} \]

\[ T_m = 1088 \text{ N-m} \]

Slip = 0.0056 \times 1088 = 6.08 \text{ rad/sec}

Slip = 58 \text{ r.p.m.}

Speed = 750 – 58 = 692 \text{ r.p.m.}

(b) Flywheel acclerating (Off Load Period)

\[ T_m = T_0 + (T_m - T_0) e^{-\frac{t}{JK}} \]

\[ T_0 = \text{No load torque} = 280 \text{ N-m} \]

\[ T_m = 1088 \text{ N-m} \quad (T_m \text{ at the beginning of the period i.e. the motor torque at the instant when load is removed}) \]

After 30 sec.,

\[ T_m = 280 + (1088 - 280) e^{-0.085 \times 30} \]

\[ T_m = 343 \text{ N-m} \]

\[ \therefore \text{Slip at this } T_m = 0.0056 \times 343 = 1.92 \text{ rad/sec} = 18.34 \text{ r.p.m} \]

\[ \therefore \text{Speed} = (750 - 18.34) \text{ r.p.m.} = 731.6 \text{ r.p.m.} \]

(ii) During 2nd cycle:

(a) Flywheel decelerating \( T_0 = 343 \text{ N-m} \).

\[ T_m = 1900 - (1900 - 343) e^{-0.085 \times 30} \]

\[ T_m = 1235 \text{ N-m} \]

\[ \therefore \text{Slip at this } T_m = 0.0056 \times 1235 = 6.92 \text{ rad/sec} \]

\[ = 66 \text{ r.p.m.} \]

\[ \rightarrow \text{speed} = 750 - 66 = 684 \text{ r.p.m.} \]

(b) Off Load Period:

\[ T_m = 280 + (1235 - 280) e^{-0.085 \times 30} \]

\[ = 354.6 \text{ N-m} \]

\[ \therefore \text{Slip at this } T_m = 0.0056 \times 354.6 = 1.99 \text{ rad/sec} \]

\[ = 19 \text{ r.p.m.} \]

\[ \text{speed} = 750 - 19 = 731.0 \text{ r.p.m.} \]

(iii) During 3rd Cycle:

(a) On Load period : \( T_m \) can be found as above.

\[ T_m = 1263 \text{ N-m} \]

\[ \text{Speed} = 683.6 \text{ r.p.m.} \]

(b) Off Load period

\[ T_m = 354.6 \text{ N-m} \]

\[ \text{speed} = 731.0 \text{ r.p.m.} \]

Initial condition at the beginning of the 3rd peak load are thus practically the same as that at the beginning of 2nd. Therefore Motor Torque in this and all succeeding load cycles will follow a similar curve to that in second period.
Example 45.19. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no-load period, the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine moment of inertia of flywheel. The no-load speed of motor is 500 r.p.m. and it has a slip of 10% on full load.

Solution.

\[ T_m = T_L - (T_L - T_0) e^{-\frac{t_0}{T}} \]
\[ T_m = T_L \left(1 - e^{-\frac{t_0}{T}}\right), \quad \because T_0 = 0 \text{ kg-m} \]

\[ T_m = 85 \text{ kg-m}, \quad T_L = 150 \text{ kg-m}, \quad T_0 = 0 \text{ kg-m}, \quad t = 15 \text{ sec.}, \quad I = ?, \quad g = 9.81 \text{ m/sec}^2 \]
\[ s = K T_{F.L.}, \quad \text{where} \quad s = \omega_0 - \omega \]
\[ \frac{2\pi (500) \times 0.1}{60} = K \times 85 \Rightarrow K = 0.0617 \]
\[ \therefore 85 = 150 \left(1 - e^{-\frac{15 \times 9.81}{60 \times 0.627}}\right); \quad \therefore I = 2884 \text{ kg-m}^2 \]

Example 45.20. A 3-φ, 50 KW, 6 pole, 960 r.p.m. induction motor has a constant load torque of 300 N-m and at wide intervals additional torque of 1500 N-m for 10 sec. Calculate

(a) The moment of inertia of the flywheel used for load equalization, if the motor torque is not to exceed twice the rated torque.

(b) Time taken after removal of additional load, before the motor torque becomes 700 N-m.

Solution.

(a) \[ P = T \times \omega \quad \therefore T = P / \omega \]
\[ T_{F.L.} = \frac{50 \times 10^3}{2\pi \times 960} = 497.36 \text{ N-m.} \]
\[ \therefore T_m = 2 \times T_{F.L.} = 2 \times 497.36 = 994.72 \text{ N-m} \]
\[ T_L = 1500 + 300 = 1800 \, \text{N-m} \]

\[ N_s = \frac{120 \, f}{P} = 1000 \, \text{r.p.m.} \]

\[ \text{F.L. slip = } 1000 - 960 = 40 \, \text{rpm} = 40 \, \text{r.p.m.} \]

\[ s = K \frac{T_{F.L.}}{2\pi(40)} = K \times 497.36 \quad \therefore \quad K = 8.42 \times 10^{-3} \]

\[ T_m = T_L - (T_L - T_0) e^{-t/K} \] as torque is in N-m

\[ 994.72 = 1800 - (1800 - 300) e^{-t/K} \]

\[ I = 1909 \, \text{kg-m}^2 \]

\( I = 8.87 \, \text{sec.} \)

**Example 45.21.** A 3-phase, 8 pole, 50 c.p.s. Induction Motor equipped with a flywheel supplies a constant load torque of 100 N-m and at wide intervals an additional load torque of 300 N-m for 6 sec. The motor runs at 735 r.p.m., at 100 N-m torque. Find moment of inertia of the flywheel, if the motor torque is not to exceed 250 N-m.

**Solution.**

\[ T_0 = 100 \, \text{N-m} \]

\[ N_s = \frac{120 \, f}{P} = 750 \, \text{rpm.} \]

\[ \text{Slip at 100 N-m torque } = 750 - 735 = 15 \, \text{r.p.m.} \]

\[ s = K \frac{T_m}{2\pi(15)} = K \times 100 \quad \therefore \quad K = 0.0157 \]

\[ T_m = T_L - (T_L - T_0) e^{-t/K} \]

\[ 700 = 300 + (994.72 - 300) e^{-t/K} \]

\[ I = 552 \, \text{kg-m}^2 \]

**Example 45.22.** A 6 pole, 50 Hz, 3-φ wound rotor Induction Motor has a flywheel coupled to its shaft. The total moment of inertia is 1000 kg-m². Load torque is 1000 N-m for 10 sec. followed by a no load period which is long enough for the motor to reach its no – load speed. Motor has a slip of 5% at a torque of 500 N-m. Find (a) Maximum torque developed by motor (b) Speed at the end of deacceleration period. [Nagpur University Winter 1996]

**Solution.**

(a) Maximum torque developed by motor

\[ T_m = T_L \left(1 - e^{-t/K}\right) \]
Rating and Service Capacity

\[ s = KT_m \]

But \( N_s = \frac{120f}{P} = 1000 \text{ r.p.m.} \)

\[ \frac{2\pi}{60} (1000 \times 0.05) = K (500) ; \quad K = 6.2 \times 10^{-3} \]

\[ T_m = 1000 \left(1 - e^{-10} \right) \quad ; \quad T_m = 796.39 \text{ N} \cdot \text{m} \]

\( (b) \quad s = KT_{F.L.} \)

\[ \frac{2\pi}{60} (100 - N) = 6.2 \times 10^{-3} \times 790.39 \]

\[ I = 277.44 \text{ kg} \cdot \text{m}^2 \]

Example 45.23. A motor fitted with a flywheel supplies a load torque of 1000 N-m for 2 sec. During no load period, the flywheel regains its original speed. The motor torque is to be limited to 500 N-m. Find moment of inertia of the flywheel. No load speed of the motor is 500 r.p.m. and its full load slip is 10%.

Solution.

\[ s = KT_{F.L.} \]

\[ \frac{2\pi}{60} (500 \times 0.1) = K 500 ; \quad K = 0.0104 \quad ; \quad T_m = T_{F.L.} \left(1 - e^{-\frac{1}{K}} \right) \]

\[ 500 = 1000 \left(1 - e^{-\frac{2}{10.0104}} \right) \quad ; \quad I = 277.44 \text{ kg} \cdot \text{m}^2 \]

Tutorial Problem No. 45.1

1. A motor driving a colliery winding equipment has to deliver a load rising uniformly from zero to a maximum of 1500 KW in 20 sec. during the accelerating period, 750 KW for 40 sec. during the full speed period and during the deceleration period of 10 sec., when regenerative braking is taking place from an initial value of 250 KW to zero and then a no load period of 20 sec. Estimate remittable KW rating of the motor. \[ 648 \text{ KW} \]

2. A constant speed drive has the following duty cycle:
   - Load rising from 0 to 400 KW – 5 minutes
   - Uniform load of 400 KW – 5 minutes
   - Regenerative power of 400 KW returned to supply – 4 minutes
   - Remains idle for – 2 minutes
   - Estimate power rating of motor. \[ 380 \text{ H. P.} \]

3. Determine the rated current of a transformer for the following duty cycle:
   - 500 A for 3 minutes
   - Sharp increase to 1000 A and constant at this value for 1 minute
   - Gradually decreasing to 200 A for 2 minutes
   - Constant at this value for 2 minutes
   - Gradually increasing to 500 A during 2 minutes repeated indefinitely. \[ 540 \text{ A} \]

4. An induction motor has to perform the following duty cycle:
   - 75 KW for 10 minutes, No load for 5 minutes
   - 45 KW for 8 minutes, No load for 4 minutes
which is repeated indefinitely.

Determine suitable capacity of a continuously rated motor. [70 H. P.]

5. A 25 H.P. motor has heating time constant of 90 min. and when run continuously on full load attains a temperature of 45°C above the surrounding air. What would be the half hour rating of the motor for this temperature rise, assuming that it cools down completely between each load period and that the losses are proportional to square of the load. [47 H.P.]

6. At full load of 10 H.P., temperature rise of a motor is 25°C after 1 hr. and 40°C after 2 hrs. Find (a) Heating time constant of motor, (b) Final temperature rise on full load. 

7. A totally enclosed motor has a temperature rise of 20°C after half an hour and 35°C after one hour on full load. Determine temperature rise after 2 hours on full load. [54.68°C]

8. A 25 H.P., 3-φ, 10 pole, 50 c.p.s. induction motor provided with a flywheel has to supply a load torque of 800 N-m for 10 sec, followed by a no load period, during which the flywheel regains its full speed. Full load slip of motor is 4% and torque-speed curve may be assumed linear over the working range. Find moment of inertia of flywheel, if the motor torque is not to exceed twice the full load torque. Assume efficiency = 90%. [718 kg-m²]

9. A motor fitted with a flywheel has to supply a load torque of 200 kg-m for 10 sec, followed by a no load period. During the no load period, the motor regains its speed. It is desired to limit the motor torque to 100 kg-m. What should be the moment of inertia of flywheel. No load speed of motor is 500 r.p.m. and has a slip of 10% at a torque of 100 kg-m. [I = 2703 kg-m²]

10. A 50 Hz., 3-φ, 10 pole, 25H.P., induction motor has a constant load torque of 20 kg-m and at wide intervals additional torque of 100 kg-m for 10 sec. Full load slip of the motor is 4% and its efficiency is 88%. Find - (a) Moment of inertia of flywheel , if motor torque not to exceed twice full load torque. (b) Time taken after removal of additional load, before motor torque is 45 kg-m. [I = 1926 kg-m², t = 9.99 sec.]

11. Define the following terms regarding the ratings of motor :-
   (i) Continuous rating  (ii) short time rating  (iii) Intermittent rating. (Nagpur University, Summer 2004)

12. With the help of heating and cooling curves define and explain the terms :
   (i) Heating time constant  (ii) Cooling time constant. (Nagpur University, Summer 2004)

13. What do you mean by ‘load-equilisation’ it is possible to apply this scheme for reversible drive? Why? (Nagpur University, Summer 2004)

14. A motor is equipped with the flywheel has to supply a load torque of 600 N-m for 10 seconds, followed by no load period long enough for flywheel to regain its full speed. It is desired to limit the motor torque of 450 N-m. What should be moment of inertia of flywheel? the no load speed of the motor is 600 rpm and has 8% slip at a torque of 450 N-m. The speed-torque characteristics of the motor can be assumed to be a straight line in the region of interest. (Nagpur University, Summer 2004)

15. A motor has the following load cycle :
   Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p. 
   Full speed period 15-85 sec Load constant at 600 h.p. 
   Decelerating period 85-100 sec h.p. returned to line falls uniformly 200 to zero 
   Decking period 100-120 sec Motor stationary. Estimate the size the motor. (J.N. University, Hyderabad, November 2003)

16. A motor driving a load has to deliver a load rising uniformly from zero to maximum of 2000 h.p. in 20 sec during the acceleration period, 1000 h.p. for 40 sec during the full speed period and during the deceleration period of 10 sec when regenerating braking taking place the h.p. returned to the supply falls from 330 to zero. The interval for decking before the next load cycle starts is 20 sec. Estimate the h.p. Rating of the motor. (J.N. University, Hyderabad, November 2003)

17. Draw and explain the output vs. time characteristics of any three types of loads. (J.N. University, Hyderabad, November 2003)
18. Discuss series and parallel operation of series and shunt motors with unequal wheeldiameters. Comment on the load sharing in each case. *(J.N. University, Hyderabad, November 2003)*

19. Discuss the various factors that govern the size and the rating of a motor for a particular service. *(J.N. University, Hyderabad, April 2003)*

20. A motor has to deliver a load rising uniformly from zero to a maximum of 1500 Kw in 20 sec during the acceleration period, 1,000 Kw for 50 sec during the full load period and during the deceleration period of 10 sec when regenerative braking takes place the Kw returned to the supply falls from an initial value of 500 to zero uniformly. The interval for decking before the next load cycle starts is 20 sec. Estimate the rating of the motor. *(J.N. University, Hyderabad, April 2003)*

21. Derive an expression for the temperature rise of an equipment in terms of the heating time constant. *(J.N. University, Hyderabad, April 2003)*

22. At full load of 10 h.p., the temperature rise of a motor is 25 degree C after one hour, and 40 degree C after 2 hours. Find the final temperature rise on full load. Assume that the iron losses are 80% of full load copper losses. *(J.N. University, Hyderabad, April 2003)*

23. Explain what you mean by Lord Equalization and how it is accomplished. *(J.N. University, Hyderabad, April 2003)*

24. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has a slip of 10% on full load. *(J.N. University, Hyderabad, April 2003)*

25. Discuss the various losses that occur in magnetic conductors which cause the temperature rise in any electrical apparatus and suggest how they can be reduced. *(J.N. University, Hyderabad, April 2003)*

26. The outside of a 12 h.p. (metric) motor is equivalent to a cylinder of 65 cms diameter and 1 meter length. The motor weighs 400 Kg and has a specific heat of 700 Joules per kg per degree C. The outer surface is capable of heat dissipation of 12 W per meter square per degree C. Find the final temperature rise and thermal constant of the motor when operating at full load with an efficiency of 90%. *(J.N. University, Hyderabad, April 2003)*

27. “A flywheel is not used with a synchronous motor for load equalization”. Discuss. *(J.N. University, Hyderabad, April 2003)*

28. A 25 h.p. 3-phase 10 pole, 50 Hz induction motor fitted with flywheel has to supply a load torque of 750 Nw-m for 12 sec followed by a no load period during which the flywheel regains its original speed. Full load slip of the motor is 4% and the torque-speed curve is linear. Find the moment of inertia of the flywheel if the motor torque is not to exceed 2 times the full load torque. *(J.N. University, Hyderabad, April 2003)*

29. Explain what do you mean by Load Equalization and how it is accomplished. *(J.N. University, Hyderabad, April 2003)*

30. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has a slip of 10% on full load. *(J.N. University, Hyderabad, April 2003)*

31. A 100 hp motor has a temperature rise of 50°C when running continuously on full load. It has a time constant of 90 minutes. Determine 1/2 hr rating of the motor for same temperature rise. Assume that the losses are proportional to the square of the load and motor cools completely between each load period. *(J.N. University, Hyderabad, December 2002/January 2003)*

32. Explain ‘load equalisation’. How this can be achieved in industrial drives. *(J.N. University, Hyderabad, December 2002/January 2003)*
33. Obtain the expression for temperature rise of a electrical machine. State the assumptions made if any.  
   \textit{(J.N. University, Hyderabad, December 2002/January 2003)}

34. A 75 kW, 500 rpm dc shunt motor is used to drive machinery for which the stored energy per kW is 5400 Joules. Estimate the time taken to start the motor, if the load torque is equal to full load torque during the starting period and the current is limited to 1 1/2 times the full load current.  
   \textit{(J.N. University, Hyderabad, December 2002/January 2003)}

### OBJECTIVE TESTS – 45

1. Heat dissipation is assumed proportional to  
   (a) Temperature difference  
   (b) Temperature difference between motor and cooling medium  
   (c) Temperature of cooling medium

2. Temperature of cooling medium is assumed  
   (a) constant  
   (b) variable

3. When the motor reaches final temperature rise its temperature remains  
   (a) constant  
   (b) falls  
   (c) rises.

4. For intermittent load, a motor of smaller rating can be used  
   (a) true  
   (b) false

5. If motor is disconnected from supply, final temperature reached will be the ambient temperature  
   (a) true  
   (b) false

6. Final temperature rise is theoretically attained only after  
   (a) fixed time  
   (b) variable time  
   (c) infinite time

7. Motor is derated when taken at altitude  
   (a) Yes  
   (b) No

8. The rolling mill load  
   (a) is constant  
   (b) fluctuates widely within long intervals of time  
   (c) fluctuates widely within short intervals of time  
   (d) varies

9. Size of motor is decided by  
   (a) load  
   (b) current  
   (c) heat produced in motor  
   (d) torque

10. Tariff is affected by sudden load drawn by motor  
    (a) true  
    (b) false

11. Flywheel helps in smoothing only  
    (a) speed fluctuations  
    (b) current fluctuations  
    (c) both of the above

12. To use flywheel, motor should have  
    (a) constant speed characteristics  
    (b) drooping speed characteristics  
    (c) variable speed characteristics

13. During light load period  
    (a) flywheel absorbs energy  
    (b) flywheel gives up energy  
    (c) flywheel does nothing

14. During peak load periods  
    (a) flywheel absorbs energy  
    (b) flywheel gives up energy  
    (c) flywheel does nothing

15. Large size of flywheel  
    (a) can be used practically  
    (b) can’t be used practically

### ANSWERS

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